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Loss of quantum coherence in a system coupled to a zero-temperature environment

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Abstract. We discuss the influence of a zero-temperature environment on a coherent quantum system. First, we calculate the reduced density operator of the system in the framework of the well-known, exactly solvable model of an oscillator coupled to a bath of harmonic oscillators. Then, we propose the sketch of an Aharonov–Bohm-like interferometer showing, through interference measurements, the decrease of the coherence length of the system due to the interaction with the environment, even in the zero temperature limit.

PACS. 03.65.Yz Decoherence; open systems; quantum statistical methods

1 Introduction

The effects of the interaction between a quantum system and its environment have been studied since the early days of quantum mechanics. For instance, in quantum measurement theory, a study of the role of the environment helps to understand the transition between the quantum and the classical world (see [1]). Another example is quantum electrodynamics (see [2]), where to some extent one can consider the electromagnetic field as the environment, influencing a charged particle (the quantum system).

Clearly, these issues are of key importance in mesoscopic physics. Since the discovery of mesoscopic phenomena in solids [3], it is well-known that the transport properties of small metallic systems at low temperatures are strongly influenced by interference of electronic waves. Examples are the weak localisation correction to conductivity or the universal sample-to-sample fluctuations of the conductance. On the other hand, the quantum behaviour of “free” electrons in mesoscopic systems is affected by their interaction with the environment, which for example consists of other electrons, phonons, photons, or scatterers. Which environment dominates the destruction of interference phenomena, an effect sometimes referred to as decoherence, depends in general on the temperature. For instance, the temperature dependence of the weak-localisation correction to the conductivity reveals that in metals electron-electron interactions dominate over the phonon contribution to decoherence at the lowest temperatures.

In this connection, the question as to what happens to interference phenomena at zero temperature has been hotly debated over the past few years. This debate was initiated by temperature-dependent weak localisation measurements [4], reporting on a residual decoherence in metals at zero temperature, in contradiction with theoretical predictions [5]. The subsequent theoretical debate [6–8] mainly focused on zero-temperature decoherence induced by Coulomb interactions in disordered electron systems, but as a spin-off has led to the more general question “Can a zero-temperature environment induce decoherence?”. Recently, this issue was discussed in [11,12] in the framework of a well-known, simple model: a harmonic oscillator (the “particle”) coupled to a chain of harmonic oscillators (the “environment”). It was shown that the particle exchanges energy with the environment, even at zero temperature. The effect of these energy fluctuations cannot be simply captured through a renormalisation of the particle’s parameters, but will give rise to a ground-state with non-trivial dynamics. This can have important consequences on thermodynamic properties of measured systems; an example is the suppression of the zero-temperature persistent normal or super-current in mesoscopic rings [9,16].

In the present paper, we are interested in the influence of an environment at low temperature on the behaviour of a mesoscopic system. We are in particular interested in the effects of the environment the interference phenomena, in the limit when the coupling energy between a small system and the environment is larger than the thermal energy. In the following, we consider a simple, exactly solvable model of a particle coupled to the environment. We regard the particle as part of a larger system of a particle coupled to a heat bath and calculate the exact reduced density operator of the particle. Then we propose the sketch of a device showing the saturation of the coherence length of the particle with decreasing temperature. The coherence length is investigated in the device by an Aharonov-Bohm interference measurement.

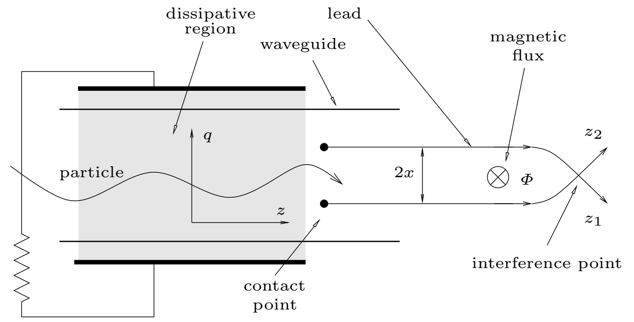

Fig. 1. A particle travelling longitudinally in the waveguide, while transversely coupled to the environment is probed by two leads.

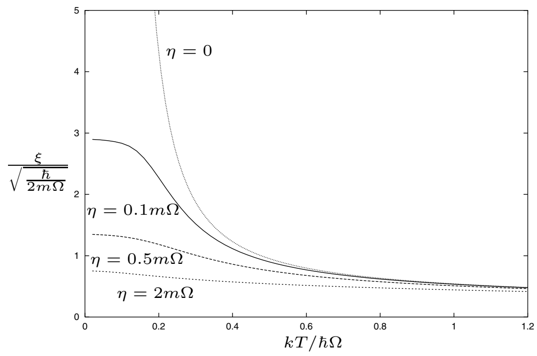

Fig. 2. Coherence length ξ as function of the temperature T for several strengths η of the coupling of the particle and the environment. The main effect is the saturation of $\xi(T)$ at finite value, when $T \to 0$.

This topic have already been discussed nearly 30 years ago in [10] where it has been shown that interference patterns in similar devices are blurred out. In our paper we present the "reduced resolvent" approach to treat effects induced by the environment. We also focus on the precise description of the interference device and the environment.

The model, largely inspired by [11,17,18], is a single particle moving longitudinally in a perfect waveguide while being transversely coupled to a continuous set of independent oscillators (the "environment"), see Figure 1. We probe the state of the particle in the waveguide with two perfect 1-dimensional leads in order to observe Aharonov-Bohm interferences; the contrast of the interference fringes is related to the transverse coherence length of the particle in the waveguide. The model is simple enough to do all computations without approximations. We will see that the coupling η between the particle and the environment destroys the interference fringes and thus reduces the coherence length ξ of the particle (Fig. 2). These effects remain even in the zero temperature limit.

This paper is organised as follows: in Section 2, we consider the effects induced on a little system by the environment; in Section 3, we present the Aharonov-Bohm-like interferometer capable of showing that the coherence length of the particle saturates in the zero-temperature limit. The calculations are worked out in Appendices A and B.

2 A particle coupled to the environment

2.1 The model

In order to get exact results, we will use the simplest well-known, non-trivial model for a particle coupled to an environment: a harmonic oscillator coupled to a set of N independent harmonic oscillators [19]. Consider the classical hamiltonian of the whole system:

$$H(q,p,\varphi,\pi) = \underbrace{\frac{p^2}{2m} + \frac{m\Omega^2}{2}q^2}_{\text{particle}} + \underbrace{\sum_{i=1}^{N}\left\{\frac{\pi_i^2}{2\mu_i} + \frac{\mu_i\omega_i^2}{2}(\varphi_i - q)^2\right\}}_{\text{environment}}. \quad (1)$$

The first part represents the "particle" (of mass m, frequency Ω, position q and momentum p). The second term corresponds to a set of N independent harmonic oscillators, ω_i is the frequency of the ith oscillator, μ_i, π_i and φ_i are its mass, momentum and position respectively. Parameters μ_i and ω_i characterise entirely the environment. One can see that in the zero-temperature limit, the energy of the whole system is zero and, therefore, the particle is at rest, i.e. is in its classical ground state. Later, we will see that this statement is no longer correct in quantum mechanics.

We will be interested in the behaviour of the system when N is large, particularly in the continuum limit. In this case, ω is a continuous variable and the mass distribution μ_i is a smooth function $\mu(\omega)$ defined such that $\mu(\omega)\,\mathrm{d}\omega$ is the mass of the oscillators with frequency between ω and $\omega + \mathrm{d}\omega$; the distribution $\mu(\omega)$ characterises entirely the environment[1]. In that case, one can prove that for the particular case $\mu(\omega) = 2\eta/m\omega^2$, the hamiltonian leads to the well-known classical equation of motion:

$$m\ddot{q} = -m\Omega^2 q \underbrace{- \eta\dot{q} + F(t)}_{\text{env.}}. \quad (3)$$

The environment induces a dissipative force $-\eta\dot{q}$ and a fluctuating force $F(t)$. The parameter $\eta/m\Omega$ defines the strength of the coupling. For that particular environment, $F(t)$ is a white noise. For other functions $\mu(\omega)$, the environment may induce an equation of motion with non-locality in time and coloured noise. In this section, we will consider any distribution $\mu(\omega)$.

Historically, the classical equation above has been derived by Lamb in 1900. Since then it has been studied in the quantum case; we give [18,19] as an entry point in the huge literature available about this topic.

[1] Often, instead of $\mu(\omega)$ we use the spectral function of the environment [17,19], defined by:

$$J(\omega) = \pi\omega^3\mu(\omega) \quad (2)$$

2.2 Reduced density operator of the particle

We consider now the quantum version of the system defined by equation (1), i.e. a quantum oscillator coupled to a quantum environment. We replace the classical variables by canonical operators:

$$q, p, \varphi, \pi, \ldots \longrightarrow \hat{q}, \hat{p}, \hat{\varphi}, \hat{\pi}, \ldots$$

We suppose that the entire system is in equilibrium at temperature T, its density operator is the following:

$$\hat{\rho} = \frac{1}{Z} e^{-\hat{H}/kT} \quad \text{where} \quad Z = \operatorname{tr} \hat{\rho}. \tag{4}$$

Generally speaking for a coupled system, $\hat{\rho}$ cannot be represented by a tensor product of a state of the particle and a state of the environment. The reduced density operator $\hat{\sigma}$ of the particle is defined as the trace over the environment of $\hat{\rho}$:

$$\hat{\sigma} = \operatorname*{tr}_{\text{env.}} \hat{\rho} \tag{5}$$

$\hat{\sigma}$ completely describes the state of the particle in the sense that it predicts any measurement made on it. As shown in appendix A, we can write $\hat{\sigma}$ in a canonical form with unknown coefficients $\tilde{T}$ and $\tilde{m}$:

$$\hat{\sigma} = \frac{1}{\tilde{Z}} \exp\left(-\frac{1}{k\tilde{T}} \hat{H}_{\text{eff}}\right) \tag{6}$$

where $\hat{H}_{\text{eff}}$ is the effective hamiltonian defined by:

$$\hat{H}_{\text{eff}} = \frac{\hat{p}^2}{2\tilde{m}} + \frac{\tilde{m}\Omega^2}{2} \hat{q}^2 \tag{7}$$

$\tilde{T}$ can be interpreted as an effective temperature a priori different from real temperature T because of the coupling with the environment. Similarly, the effective mass $\tilde{m}$ differs from the mass m of the particle. In particular we will see that even at zero temperature $T = 0$, we may have $\tilde{T} > 0$. Therefore, $\hat{\sigma}$ is not a pure state, in particular, the particle cannot be in its ground state. Thus, parameters $\tilde{T}$ and $\tilde{m}$ necessarily differ from the real temperature T and the real mass m of the particle. $\tilde{Z}$ is a normalisation constant that ensures that $\operatorname{tr} \hat{\sigma} = 1$. We can explicitly calculate $\tilde{T}$ and $\tilde{m}$ as function of the real temperature T and the mass distribution $\mu(\omega)$ of the environment (see Appendix A).

The definition of $\tilde{T}$, $\tilde{m}$ and $\hat{H}_{\text{eff}}$ that we followed is not standard. Since only two of the coefficients $\tilde{T}$, $\tilde{m}$ and Ω are independent, we have chosen to avoid having an effective frequency $\tilde{\Omega}$. With this definition the effective hamiltonian has the same spectrum as the hamiltonian of the particle[2]. Also, keeping Ω the same in both hamiltonians is the only way to ensure that there is a linear canonical transformation of particle hamiltonian leading to $\hat{H}_{\text{eff}}$. With this requirement, parameters $\tilde{T}$ and $\tilde{m}$ are uniquely defined. Note the only relevant intrinsic quantity describing the particle is the reduced density operator $\hat{\sigma}$.

[2] Note that the effective hamiltonian $\hat{H}_{\text{eff}}$ does not generate the (non–unitary) time evolution of the dissipative particle.

This study remains true for any function $\mu(\omega)$; however if there is a large gap in $\mu(\omega)$ around Ω, the reduced resolvent (discussed in Appendix A) can have a pole on the real axis. It corresponds to a discrete oscillating mode, analogue of a dressed state in atomic physics. Because of the coupling between particle and environment, this mode has components on both the particle and the environment. If it is initially excited then it will oscillate forever and the particle will never reach equilibrium. It has been shown by Grabert et al. [14] that if:

$$\operatorname*{tr}_{\text{part}} \hat{\rho}(t=0) = \frac{e^{\hat{H}_{\text{env}}/kT}}{Z_{\text{env}}} \tag{8}$$

and $\mu(\omega)$ has no gap, then:

$$\lim_{t\to\infty} \operatorname*{tr}_{\text{env}} \hat{\rho}(t) = \hat{\sigma}. \tag{9}$$

This result can also be obtained [20] by the "reduced resolvent approach". In Section 3, we will suppose that $\mu(\omega)$ is such that the reduced resolvent of the system has no poles on the real axis.

2.3 An example — ohmic environment

In order to illustrate the results discussed above, consider the case of an ohmic environment, defined by:

$$\mu(\omega) = \begin{cases} 2\eta/\pi\omega^2 & \text{if} \quad \omega < \Omega_c \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

In this case, for $\Omega_c \gg \Omega$, the classical behaviour of the particle is described by equation (3), see [11,17,18]. Note that we don't want to take the limit $\Omega_c \to \infty$ because the cut-off frequency Ω_c has a physical meaning; in the example we consider in the next section, it is directly related to parameters of the environment (see Sect. 3.2). $\tilde{T}(T)$ can be calculated explicitly (see Appendix A). It is plotted for different values of the coupling η between the particle and the environment in Figure 3. At $T = 0$, we find the following limit expression for small η:

$$k\tilde{T}(0) \sim \frac{\hbar\Omega}{2} \times \frac{2}{\ln(2\pi/\gamma)} \qquad \text{where} \quad \gamma = \frac{\eta}{m\Omega} \ln(\Omega_c/\Omega). \tag{11}$$

We can see that, even at $T = 0$, the behaviour of the particle is very similar to the behaviour of a particle at strictly positive temperature: its reduced density operator is not a pure state as it would be for an isolated particle, but a statistical mixture.

Similarly $\tilde{m}(T)$ can be calculated explicitly (see Appendix A). It is plotted for different values of η in Figure 4. At $T = 0$, we find the following limit expressions, for small η:

$$\tilde{m}(0) \sim m\,(1 + \gamma/\pi). \tag{12}$$

It is interesting to evaluate the entropy of the particle, defined by $S = -\operatorname{tr}(\hat{\sigma} \ln \hat{\sigma})$. In contrast to $\tilde{T}$ and $\tilde{m}$, S

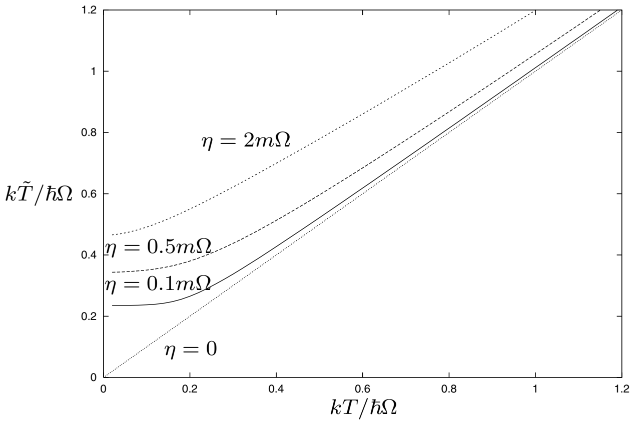

Fig. 3. Effective temperature $\tilde{T}$ as function of the temperature T, plotted for different values of the coupling η.

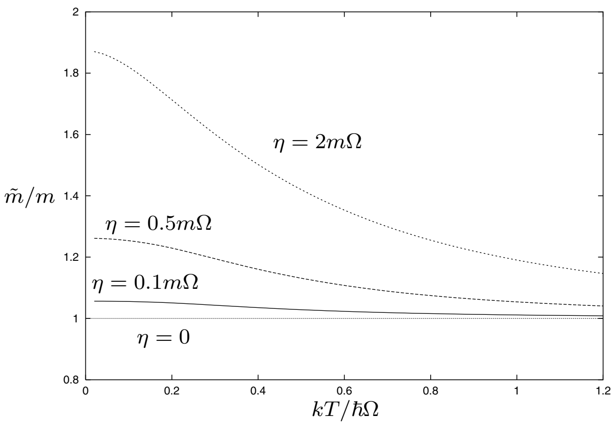

Fig. 4. Effective mass $\tilde{m}$ as function of the temperature T, plotted for different values of the coupling η.

has an intrinsic definition. In Figure 5, we see that there is a residual entropy that does not vanish at $T = 0$. These statements are not really in contradiction with the ordinary statistical mechanics (or with the 3-rd principle of thermodynamics); indeed in statistical mechanics one neglects the coupling energy between the particle and the reservoir. That approximation is good at high temperature when $T \gg \tilde{T}(0)$.

Note that all results in this section are well known for at least 25 years [11,13,19], we present them in order to use them in the next section. The "reduced resolvent method" that we used to obtain them (Appendix A) is common in spectral theory [23] but to our knowledge it haven't been applied to this particular problem yet.

3 Interferences in a device

In order to illustrate the physical meaning of these results, in the following section we propose the sketch of an Aharonov-Bohm interferometer whose purpose is to measure the spatial coherence length ξ of the charged particle. We will see that its coherence length decreases as the coupling between the particle and the environment increases, even at zero temperature.

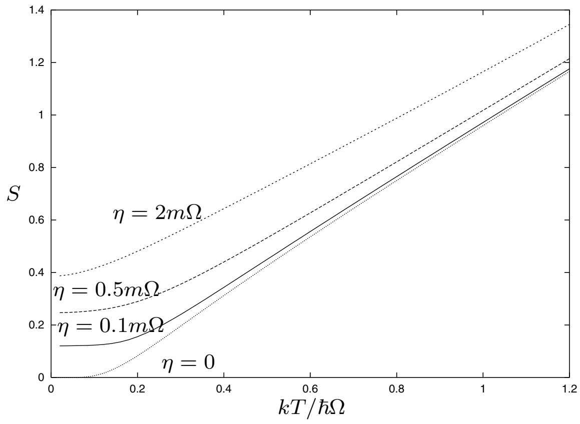

Fig. 5. Entropy S of the particle as function of the temperature T, plotted for different values of the coupling η.

3.1 Description of the device

Consider a perfect two-dimensional waveguide. In the longitudinal direction (z-axis) the particle is free, while in the transverse direction (q-axis) it is confined by a harmonic potential (see Fig. 1). Hence the hamiltonian of the particle in the waveguide is:

$$\hat{H}_{\text{guide}} = \underbrace{\frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2}\hat{q}^2}_{\hat{H}_{\text{transv.}}} + \frac{\hat{p}_z^2}{2m}. \tag{13}$$

We furthermore consider that at $z = 0$ are connected two 1-dimensional leads separated by a distance $2x$, as shown in Figure 1. A particle moving in the waveguide can enter the "upper" one or the "lower" one. In lead 1 (2) the particle is described as one-dimensional, moving along the z_1-axis (z_2-axis):

$$\hat{H}_{\text{lead}_i} = \frac{\hat{p}_{z_i}^2}{2m}. \tag{14}$$

The Hilbert space of the particle is the direct sum of the space of the waveguide and the space of the leads [21].

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{guide}} \oplus \mathcal{H}_{\text{lead1}} \oplus \mathcal{H}_{\text{lead2}}. \tag{15}$$

Thus, a quantum state of the particle is given by writing the state of the particle in these three spaces:

$$\begin{pmatrix} |\psi\rangle_{\text{guide}} \\ |\psi\rangle_{\text{lead1}} \\ |\psi\rangle_{\text{lead2}} \end{pmatrix}. \tag{16}$$

We suppose that the coupling between each lead and the waveguide corresponds to a "tunnel" coupling. The wire and the waveguide interact only in a very localised region. The total hamiltonian (coupling included) can be written as:

$$\hat{H}_{\text{tot}} = \begin{pmatrix} \hat{H}_{\text{guide}} & \hat{V}_1 & \hat{V}_2 \\ \hat{V}_1^+ & \hat{H}_{\text{lead1}} & 0 \\ \hat{V}_2^+ & 0 & \hat{H}_{\text{lead1}} \end{pmatrix}$$

$$\hat{V}_1 = \alpha|v_1\rangle\langle f_1|$$

$$\hat{V}_2 = \alpha|v_2\rangle\langle f_2|.$$

The coupling energy is represented by the off-diagonal terms. The states $|f_1\rangle$ and $|f_2\rangle$ of the lead are supposed to be very localised near the leads' origin. The states in the waveguide $|v_1\rangle$ and $|v_2\rangle$ are localised near the contact points $z = 0, q = x$ and $z = 0, q = -x$. The real constant α represents the strength of the coupling. When α vanishes, the tunnelling between the waveguide and the leads disappears. By putting a variable flux Φ between the two leads, one can induce a phase shift ϕ between them. The probability to detect the particle as a function of the flux oscillates by varying the flux (see Fig. 1). This Aharonov-Bohm interferometer [22] is nearly equivalent to the Young double slit.

3.2 Transverse coupling with the environment

Now, let us suppose that there is a region in the waveguide where the particle is *transversely* coupled with an external environment (for instance, through a capacitor for a charged particle), see Figure 1. In the case of a linear resistor and a perfect capacitor, it is straightforward (but tedious) to show that the spectral function $\mu(\omega)$ of the environment is:

$$\mu(\omega) = \frac{2e^2}{\pi\omega^2 l^2 C} \times \frac{1/RC}{\omega^2 + (1/RC)^2} \tag{17}$$

where e is the charge of the particle, l is the distance between the plates of the capacitor C is the capacitance and R the resistance of the linear resistor. For large R, the environment has typical strength $\eta = Re^2/l^2$ and cut-off frequency $\Omega_c = 1/RC$. Finally we should note that the transversal frequency of the waveguide is renormalised by the capacitive coupling as follows:

$$m\Omega^2 = m\Omega^2_{\text{guide}} - \frac{e^2}{l^2 C} \tag{18}$$

where Ω_{guide} is the frequency of the guide without coupling.

The environment acting in the q-direction is represented by a linear resistor, and is well described by the model considered in Section 2. We suppose that the dissipative region is large enough in the z-direction so that any particle entering on the left will have reached thermal equilibrium with the bath in the q-direction before leaving it on the right. It means that if a particle enters the region with a pure state $|\psi\rangle \otimes |k\rangle$ where $|\psi\rangle$ is any transverse state and $|k\rangle$ is a plane wave in the z-direction, it will leave the dissipative region in a state described by a density operator:

$$\hat{\sigma} \otimes |k\rangle\langle k| \tag{19}$$

where $\hat{\sigma}$ is the particle state in the q-direction, given by equation (6). We also suppose that the distance between the end of the dissipative region and the contact points of the pair of leads is much smaller than $\hbar k/m\Omega$.

More precisely, the total hamiltonian in the guide is:

$$H(q,p,\varphi,\pi) = \frac{p_z^2}{2m} + \frac{p^2}{2m} + \frac{m\Omega^2}{2}q^2 + \sum_{i=1}^{N} \left\{ \frac{\pi_i^2}{2\mu_i} + \frac{\mu_i\omega_i^2}{2}\left(\varphi_i - \lambda(z)q\right)^2 \right\}$$

where $\lambda(z) = 1$ if the particle is inside the dissipative region and $\lambda(z) = 0$ otherwise. Except on the boundaries of the dissipative region, we suppose that the coupling between particle and environment is transverse. Thus, except on the boundaries, the transverse and the longitudinal dynamics are independent. Scattering on the boundaries is possible. On the one hand, $\lambda(z)$ has no singularities, thus the dependence on z of the stationary states of the system is continuous and the state of the particle is the same on both sides of any boundary. On the other hand, one can show that we can neglect scattering by assuming that the initial longitudinal energy fulfils the following two conditions:

$$\frac{p_z^2}{2m} \gg \langle q^2\rangle \sum \mu_i\omega_i^2$$
$$\frac{p_z^2}{2m} \gg \left(\langle q^2\rangle \sum \mu_i\omega_i^2\langle\epsilon_i\rangle\right)^{1/2}$$

where $\langle\epsilon_i\rangle$ is the mean energy of the i-th oscillator of the environment. With these assumptions, longitudinal and transverse degrees of freedom are independent along the whole guide. In this case the z-coordinate, the longitudinal dynamics is trivial, it simply behaves as translation in time.

Experimentally, the initial pure state $|\psi\rangle \otimes |k\rangle$ can be prepared by putting a particle with fixed wave-vector k in a single-channelled waveguide. At this stage, the only possible transverse state is the ground state. However, if the number of transverse channels grows *adiabatically* along the z-axis, it turns out that the particle will necessarily remain in the ground state [15]. In this case $|\psi\rangle = |0\rangle$, the transverse ground state of the waveguide.

3.3 Interference fringes

In Appendix B, it is shown that in the limit when the incident wave vector satisfies the condition that its longitudinal energy is much larger than its transverse energy, the probability that the particle be found in the cross-point of the two leads, after a measure is in term of any transverse state $\hat{\sigma}$:

$$P(\phi) = |\tau|^2 \times \Big(\sigma(x,x) + \sigma(-x,-x)$$
$$\sigma(-x,x)e^{i\phi} + \sigma(x,-x)e^{-i\phi}\Big)$$

where $\sigma(x,x') = \langle x|\hat{\sigma}|x'\rangle$ is the position representation of $\hat{\sigma}$. Similarly, let P_1 and P_2 be the probability to find the particle respectively in lead 1 and lead 2:

$$P_1 = |\tau|^2\sigma(x,x)$$
$$P_2 = |\tau|^2\sigma(-x,-x).$$

We define the normalised contrast C of the interference fringes by:

$$C^2 = \frac{\langle P^2 \rangle - \langle P \rangle^2}{2P_1P_2}, \quad \text{where } \langle f \rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\,\mathrm{d}\phi. \tag{20}$$

By using the value of $\sigma(q, q')$, discussed in Appendix A:

$$\sigma(x, x') = \frac{1}{2\pi\langle \hat{q}^2 \rangle} e^{-\frac{1}{2}\left[\frac{\langle \hat{p}^2 \rangle}{\hbar^2}(x-x')^2 + \frac{1}{4\langle \hat{q}^2 \rangle}(x+x')^2\right]}$$

we obtain:

$$P(\phi) = 2|\tau|^2\sigma(x, x) \times \left(1 + e^{-x^2/2\xi^2}\cos\phi\right)$$
$$\text{where} \quad \xi^2 = \frac{\hbar}{2\tilde{m}\Omega} \times \frac{1}{1 - \mathrm{th}\,\frac{\hbar\Omega}{2k\tilde{T}}}.$$

Finally, we obtain for the normalised contrast:

$$C = \exp -\frac{x^2}{2\xi^2}. \tag{21}$$

This expression defines the transverse *coherence length* ξ of the particle. It shows that the particle is capable to interfere along a maximum length of ξ.

For the environment discussed in Section 2, consider the environment discussed in Section 2.3. We can explicitly calculate ξ for that particular environment (see Appendix B). Figure 2 shows the transverse coherence length of the particle, as function of the (real) temperature, for various values of the coupling strength η. In the zero-temperature limit, saturation occurs and $\xi(T = 0)$ has a finite value, showing that long range interferences are not possible. At $T = 0$, we find the following limit expression of ξ, for small η:

$$\xi^2(0) \sim \frac{\hbar}{\eta} \times \frac{\pi}{4\ln(\Omega_c/\Omega)}. \tag{22}$$

Even at zero temperature, the interference fringes are destroyed because of the coupling between particle and environment. The behaviour of the particle is similar to that of a particle at strictly positive temperature. This effect becomes more pronounced as the coupling strength increases.

Finally, let us remark that the evolution in the waveguide is not reversible because the environment has infinite number of degrees of freedom. This should be taken into account when transport phenomenas are considered.

4 Conclusion

We considered a simple exactly solvable model of a particle coupled to the environment. Our purpose was to discuss the influence of the environment on the particle at thermal equilibrium. After obtaining the reduced density operator of the particle, we proposed a sketch for a simple device capable to measure the spatial coherence length of the particle through Aharonov-Bohm interference measurement. We saw that the coupling between the particle and the environment destroys long range interferences and reduces its spatial coherence length. Even in the zero temperature limit, this effect remains and grows with the coupling between the particle and the environment.

We finally discuss two possible experimental realisations of our model.

Let us first consider a confined two-dimensional electron gas, where the coherence length of the electron is about several hundred nanometres, comparable to the typical size of a small mesoscopic device [28]. In view of this scenario, an external circuit giving rise to electrodynamic noise could play the role of the environment. The main problem is the construction of the point contact between the leads and the waveguide. Actually, if the contact is very localised, one may loose a significant part of the measured signal. Furthermore, a direct interpretation of such an experiment could be masked by many-body effects: the Pauli principle has to be taken into account when treating decoherence in a two-dimensional electron gas, along the lines presented in [26].

Another example is a cold atomic gas, localised in a magnetic trap. The atomic gas is a quantum system, that looses coherence in the presence of fluctuations of the trapping potential, induced by fluctuations of the applied magnetic field (the "environment"). Interference-like experiments, similar to the ones discussed here, have been proposed to study these decoherence phenomena [27]. The environment in [27] is a high-temperature one; it would be interesting to extend the discussion to the case of a low-temperature environment, such that the effects discussed in the present paper become important.

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Appendix A: Reduced density operator of the particle

A.1 Classical modes

The classical system defined by equation 1 is made of $N+1$ harmonic oscillators coupled one to each other, but since the hamiltonian is quadratic and positively defined, one can decompose the system as a set of $N + 1$ *independent* harmonic oscillators (the eigen-modes of the total system).

A.1.1 Matrix notations

Let us first rewrite the classical hamiltonian 1 in a matrix form, where we separate the $N+1$ positions from the $N+1$ momenta:

$$H = \frac{1}{2}(P|P) + \frac{1}{2}(Q|A|Q)$$

where:

$$|P) = \begin{pmatrix} \frac{p}{\sqrt{m}} \\ \vdots \\ \frac{\pi_i}{\sqrt{\mu_i}} \\ \vdots \end{pmatrix}, \quad |Q) = \begin{pmatrix} q\sqrt{m} \\ \vdots \\ \phi_i\sqrt{\mu_i} \\ \vdots \end{pmatrix}$$

$$A = \begin{pmatrix} \Omega^2 + \sum \omega_i^2 \frac{\mu_i}{m} & \dots & -\omega_i^2 \sqrt{\frac{\mu_i}{m}} & \dots \\ \vdots & \ddots & & \\ -\omega_i^2 \sqrt{\frac{\mu_i}{m}} & & \omega_i^2 & 0 \\ \vdots & & 0 & \ddots \end{pmatrix}.$$

We note $|0), \dots, |N)$ the canonical base in $\mathbf{R}^{N+1}$, so that the vectors $|Q)$ and $|P)$ in $\mathbf{R}^{N+1}$ have components:

$$\begin{aligned} q\sqrt{m} &= (0|Q) & \pi_i/\sqrt{\mu_i} &= (i|P) \\ \varphi_i\sqrt{\mu_i} &= (i|Q) & p/\sqrt{m} &= (0|P). \end{aligned} \tag{23}$$

The eigen-vectors $|U_j)$ and eigen-values ν_j^2 of the symmetric and positive matrix A verify:

$$A|U_j) = \nu_j^2|U_j) \tag{24}$$

$$I = \sum_{j=0}^{N} |U_j)(U_j|. \tag{25}$$

We can, now, rewrite the hamiltonian with respect to a new set of "normal" positions and momenta:

$$H = \sum_{j=0}^{N} H_j = \frac{1}{2}\sum_{j=0}^{N} \left\{ y_j^2 + \nu_j^2 x_j^2 \right\} \tag{26}$$

with the new canonical coordinates $x_j = (U_j|Q)$ and $y_j = (U_j|P)$. Conversely, because $s_{ij} = (i|U_j)$ is an orthogonal matrix:

$$\begin{aligned} q\sqrt{m} &= \sum_{j=0}^{N}(0|U_j)x_j & p/\sqrt{m} &= \sum_{j=0}^{N}(0|U_j)y_j \\ \varphi_i\sqrt{\mu_i} &= \sum_{j=0}^{N}(i|U_j)x_j & \pi_i/\sqrt{\mu_i} &= \sum_{j=0}^{N}(i|U_j)y_j. \end{aligned} \tag{27}$$

A.1.2 Reduced resolvent

In Section 4, equations (41) and (42) we need to compute:

$$M_f = \sum_{j=0}^{N}(0|U_j)^2 f(\nu_j^2) \tag{28}$$

where f is any smooth function. We will follow a standard calculation, using the resolvent, to explicitly obtain that expression (see [23–25]). Consider the resolvent $R(z)$ of the matrix A, defined by as $R(z) = (z - A)^{-1}$, with $z \in \mathbf{C}$. In the eigen-base of A, $R(z)$ can be written as:

$$R(z) = \sum_{j=0}^{N} \frac{|U_j)(U_j|}{z - \nu_j^2}. \tag{29}$$

For z near the real axis, we can set $z = \epsilon + i\kappa$ and take the limit $\kappa \to 0$ in the imaginary part of R:

$$\mathrm{Im}\left\{ \lim_{\kappa\to 0^+} R(\epsilon + i\kappa) \right\} = -\pi \sum_{j=0}^{N} |U_j)(U_j|\delta(\epsilon - \nu_j^2). \tag{30}$$

Since $\nu_j^2 > 0$, only the positive ϵ are concerned and we can take $\epsilon = u^2$, we obtain:

$$M_f = -\frac{1}{\pi}\int_0^\infty \left\{ \lim_{\kappa\to 0^+} \mathrm{Im}\,(0|R(u^2 + i\kappa)|0) \right\} f(u^2)\,\mathrm{d}u^2. \tag{31}$$

Let us decompose the matrix A as the sum of its diagonal and its off-diagonal part: $A = A_0 + V$. One can write R as:

$$R = R_0 + R_0 V R, \text{ where } R_0(z) = (z - A_0)^{-1}. \tag{32}$$

Consider:

$$\begin{aligned} (0|R|0) &= (0|R_0|0) + (0|R_0|0)(0|VR|0) \\ &= (0|R_0|0)\left[1 + \sum_{i=1}^{N}(0|V|i)(i|R|0)\right] \\ &= \frac{1}{z - \Omega^2 - \sum_{i=1}^{N}\omega_i^2\mu_i/m} \\ &\quad\times \left[1 - \sum_{i=1}^{N}\omega_i^2\sqrt{\frac{\mu_i}{m}}(i|R|0)\right] \end{aligned} \tag{33}$$

$$\begin{aligned} (i|R|0) &= (i|R_0|0) + (i|R_0|i)(i|V|0)(0|R|0) \\ &= -\frac{1}{z - \omega_i^2}\,\omega_i^2\sqrt{\frac{\mu_i}{m}}\,(0|R|0). \end{aligned} \tag{34}$$

Finally, by combining the last two expressions, we obtain:

$$(0|R(z)|0)^{-1} = z - \Omega^2 - z\sum_{i=1}^{N}\frac{\omega_i^2\mu_i/m}{z - \omega_i^2}. \tag{35}$$

A.1.3 Continuum limit

In that form, the expression of $(0|R|0)$ is exact but nearly unusable. However, remember that we are mainly interested in the case where the number of oscillators in the environment is very large, so we can take the continuum limit $N \to \infty$ and replace the set of frequencies ω_i with a continuous variable ω. The mass distributions is described by the smooth function $\mu(\omega)$ defined such that $\mu(\omega)\,\mathrm{d}\omega$ is the mass of the oscillators with frequency between ω and $\omega + \mathrm{d}\omega$. In that limit,

$$(0|R(z)|0)^{-1} = z - \Omega^2 - z\int_0^\infty \frac{\omega^2\mu/m}{z - \omega^2}\,\mathrm{d}\omega. \tag{36}$$

Now, we can take $z = u^2 + i\kappa$ and consider the limit $\kappa \to 0$:

$$\lim_{\kappa\to 0^+}(0|R(u^2 + i\kappa)|0) = \frac{1}{u^2 - \Omega^2 - \Delta(u) + iu\,\Gamma(u)}$$

where:

$$\Delta(u) = \mathcal{P}\int_0^\infty \frac{\omega^2 u^2/m}{u^2-\omega^2}\mu(\omega)\,\mathrm{d}\omega$$
$$\Gamma(u) = \frac{\pi u^2}{2m}\mu(u).$$

Finally, by putting the last expression in equation 31, we obtain in the continuum limit:

$$\sum_{j=0}^{N}(0|U_j)^2 f(\nu_j^2) \longrightarrow \frac{2}{\pi}\int_0^\infty \frac{\Gamma u^2 f(u^2)\,\mathrm{d}u}{(u^2-\Omega^2-\Delta)^2+\Gamma^2u^2}. \tag{37}$$

A.2. Fluctuation of the position and the momentum of the particle

We consider now the quantum version of the system defined by equation (1), i.e. a quantum oscillator coupled to a quantum environment. We replace the classical variables with operators; in the case when the whole system is in equilibrium at temperature T, its density operator is the following:

$$\hat{\rho} = \frac{1}{Z}e^{-\hat{H}/kT} \quad \text{where} \quad Z = \mathrm{tr}\,\hat{\rho}. \tag{38}$$

From equation (26) we can write $\hat{\rho}$ as:

$$\hat{\rho} = \frac{e^{-\hat{H}_0/kT}}{Z_0}\otimes\frac{e^{-\hat{H}_1/kT}}{Z_1}\otimes\cdots\otimes\frac{e^{-\hat{H}_N/kT}}{Z_N} \tag{39}$$
$$\text{where} \;\; Z_j = \mathrm{tr}\left(e^{-\hat{H}_j/kT}\right). \tag{40}$$

The mean square fluctuations of the position of the particle in the state $\hat{\rho}$ are:

$$\begin{aligned}\langle\hat{q}^2\rangle_T &= \frac{1}{m}\langle\sum_{j=0}^{N}(0|U_j)\hat{x}_j\sum_{j'=0}^{N}(0|U_{j'})\hat{x}_{j'}\rangle\\ &= \frac{1}{m}\sum_{j=0}^{N}(0|U_j)^2\langle\hat{x}_j^2\rangle\\ &= \sum_{j=0}^{N}(0|U_j)^2\left\{\frac{\hbar}{2m\nu_j}\coth\frac{\hbar\nu_j}{2kT}\right\}.\end{aligned} \tag{41}$$

Where in the last step we have replaced $\langle\hat{x}_j^2\rangle$ with its value, obtained for a simple 1-dimensional oscillator. In the same way, we can find the mean square fluctuations of the position of the particle:

$$\langle\hat{p}^2\rangle_T = \sum_{j=0}^{N}(0|U_j)^2\left\{\frac{m\hbar\nu_j}{2}\coth\frac{\hbar\nu_j}{2kT}\right\}. \tag{42}$$

We see that these expressions have the form of equation (37), so, for a continuous environment, we can explicitly compute $\langle\hat{q}^2\rangle$ and $\langle\hat{p}^2\rangle$ for any mass distribution $\mu(\omega)$.

A.3. Reduced density operator of the particle

The reduced density operator $\hat{\sigma}$ of the particle is defined as the trace over the environment of $\hat{\rho}$:

$$\hat{\sigma} = \underset{\text{env.}}{\mathrm{tr}}\,\hat{\rho}. \tag{43}$$

Since the total hamiltonian is quadratic in position and momentum, $\hat{\rho}$ is Gaussian in these operators; when we trace over the degrees of freedom of the environment, $\hat{\sigma}$ remains Gaussian and we can write:

$$\hat{\sigma} = e^{a\hat{p}^2+b\hat{q}^2+c} \tag{44}$$

with unknown coefficients a, b, c. Note that there is not a term of the form $\hat{p}\hat{q}$ because the total hamiltonian is invariant by time reversal and therefore $\hat{\rho}$ and $\hat{\sigma}$ are also invariant. We can rewrite $\hat{\sigma}$ in a canonical form with unknown coefficients $\tilde{T}$ and $\tilde{m}$:

$$\hat{\sigma} = \frac{1}{\tilde{Z}}\exp\left\{-\frac{1}{k\tilde{T}}\left(\frac{\hat{p}^2}{2\tilde{m}}+\frac{\tilde{m}\Omega^2}{2}\hat{q}^2\right)\right\} \tag{45}$$

Let us now determine $\tilde{Z}$, $\tilde{T}$ and $\tilde{m}$. $\tilde{Z}$ is a normalisation constant that ensures that $\mathrm{tr}\,\hat{\sigma} = 1$. We can explicitly calculate:

$$\langle\hat{p}^2\rangle \overset{\text{def}}{=} \mathrm{tr}(\hat{p}^2\hat{\sigma}) = \frac{\hbar\tilde{m}\Omega}{2}\coth\frac{\hbar\Omega}{2k\tilde{T}} \tag{46}$$
$$\langle\hat{q}^2\rangle \overset{\text{def}}{=} \mathrm{tr}(\hat{q}^2\hat{\sigma}) = \frac{\hbar}{2\tilde{m}\Omega}\coth\frac{\hbar\Omega}{2k\tilde{T}}. \tag{47}$$

Conversely, we obtain:

$$k\tilde{T} = \frac{\hbar\Omega}{2}\frac{1}{\mathrm{Argth}\sqrt{\frac{\hbar^2}{4\langle\hat{p}^2\rangle\langle\hat{q}^2\rangle}}} \tag{48}$$
$$\tilde{m} = \sqrt{\frac{\langle\hat{p}^2\rangle}{\Omega^2\langle\hat{q}^2\rangle}}. \tag{49}$$

Finally, since according to equation (41), (42) and (37), $\langle\hat{q}^2\rangle$ and $\langle\hat{p}^2\rangle$ can be written as function of the mass distribution of the environment $\mu(\omega)$, we can do either for $\tilde{m}$ and $\tilde{T}$. Finally, we have obtained the expression of the reduced density operator, which, in position representation, can be written as:

$$\sigma(q,q') = \frac{1}{2\pi\langle\hat{q}^2\rangle}e^{-\frac{1}{2}\left[\frac{\langle\hat{p}^2\rangle}{\hbar^2}(q-q')^2+\frac{1}{4\langle\hat{q}^2\rangle}(q+q')^2\right]}. \tag{50}$$

These results have already been obtained following different approaches, like the fluctuation–dissipation theorem [11,13,19] and the functional integral approach [14]. The decomposition in classical modes is quite natural and conceptually simple; we think that it is interesting to show this method here also because it can be used to handle non-equilibrium situations in a very simple way. The above calculations are similar in spirit to the works of Ullersma [29] and Davies [30].

Appendix B: Interference fringes

B.1 Scattering coefficients

Consider the coupling between the waveguide and the leads, discussed in Section 3.1. We will rewrite it in the following way:

$$\hat{V}_1 = \alpha(|g_1\rangle \otimes |f\rangle)\langle f_1|$$
$$\hat{V}_2 = \alpha(|g_2\rangle \otimes |f\rangle)\langle f_2|.$$

The states $|f_1\rangle$ and $|f_2\rangle$ of the lead are supposed to be very localised near the leads' origin. The transverse states in the waveguide $|g_1\rangle$ and $|g_2\rangle$ are localised near the attach points $q = x$ and $q = -x$ and the longitudinal state $|f\rangle$ is localised around the origin $z = 0$. The real constant α represents the strength of the coupling. Note that the later states are normalised and not Dirac peaks, they are such that:

$$\langle f|f\rangle = 1$$
$$\langle z|f\rangle \sim \sqrt{\varepsilon}\ \delta(z)$$

where ϵ is the typical spatial width of the state $|f\rangle$. We are interested by the stationary state of the particle, made of an incident wave, a reflected wave and three transmitted waves (one through the waveguide and two through the pair of leads). Suppose that the incident wave is in the n'th channel (n'th excited transverse state). It may be transmitted and reflected in the other channels, we can write the stationary wave function in the nth channel and the leads as:

$$\psi_n(q, z) = \chi_n(q)\phi_n(z) \quad (51)$$

where $\chi_n(q)$ is the stationary wave-function of the transverse "harmonic oscillator" with energy $E_n = \hbar\Omega(n+1/2)$; the longitudinal part $\phi_n(z)$ of the wave-function is made by an incident, a reflected and a transmitted wave:

$$\phi_n(z) = \begin{cases} \text{if } z < 0 & \delta_{nn'}e^{ik_n z} + r_n e^{-ik_n z} \\ \text{if } z > 0 & t_n e^{ik_n z} \end{cases}$$

$\delta_{nn'}$ is the Kronecker symbol, t_n and r_n respectively are the transmission and the reflection coefficients in the nth channel and k_n is the corresponding wave-vector:

$$k_n = \sqrt{k^2 + 2m(E_{n'} - E_n)/\hbar^2}. \quad (52)$$

Finally, the wave-function in each lead is just a transmitted plain-wave, it can be written as:

$$\sigma_1(z_1) = s_1 e^{i\kappa|z_1|}$$
$$\sigma_2(z_2) = s_2 e^{i\kappa|z_2|}$$

where s_1, s_2 are the transmission coefficients in the guide and κ the corresponding wave-vector:

$$\kappa = \sqrt{k^2 + 2mE_{n'}/\hbar^2}. \quad (53)$$

Since this state is supposed to be stationary, it is an eigenvector of the hamiltonian in 3.1, we can write:

$$-\frac{\hbar^2}{2m}\sigma_1'' + \alpha\varepsilon^{3/2}\delta(z_1)\sum_{n=0}^{\infty}\chi_n(x)\phi_n(0) = E\sigma_1$$

$$-\frac{\hbar^2}{2m}\sigma_2'' + \alpha\varepsilon^{3/2}\delta(z_2)\sum_{n=0}^{\infty}\chi_n(-x)\phi_n(0) = E\sigma_2$$

$$E_n\phi_n - \frac{\hbar^2}{2m}\phi_n'' + \alpha\varepsilon^{3/2}\delta(z)\sigma_1(0)\bar{\chi}_n(x)\delta(z_1)$$
$$+\alpha\varepsilon^{3/2}\delta(z)\sigma_2(0)\bar{\chi}_n(-x)\delta(z_2) = E\phi_n.$$

These three equations, and the continuity condition on ϕ_n, leads to the following set of linear equations:

$$-\hbar^2 ik_n(t_n - \delta_{nn'} + r_n)/2m + \alpha\varepsilon^{3/2}s_2\chi_n(-x) = 0$$

$$-\hbar^2 i\kappa s_1/2m + \alpha\varepsilon^{3/2}\sum_{n=0}^{\infty}\chi_n(x)t_n = 0$$

$$-\hbar^2 i\kappa s_2/2m + \alpha\varepsilon^{3/2}\sum_{n=0}^{\infty}\chi_n(-x)t_n = 0$$

$$\delta_{nn'} + r_n - t_n = 0 \quad (54)$$

finally, we obtain:

$$s_1 = \frac{\alpha\varepsilon^{3/2}m}{i\hbar^2\kappa}\frac{1}{R^2 + |Z|^2}[R\chi_{n'}(x) - Z\chi_{n'}(-x)]$$

$$s_2 = \frac{\alpha\varepsilon^{3/2}m}{i\hbar^2\kappa}\frac{1}{R^2 + |Z|^2}[R\chi_{n'}(-x) - \bar{Z}\chi_{n'}(x)]$$

where:

$$R = 1 + \frac{\alpha^2\varepsilon^3 m^2}{\hbar^2\kappa^2}\sum\frac{1}{\lambda_n}|\chi_n(x)|^2 \quad (55)$$

$$Z = \frac{\alpha^2\varepsilon^3 m^2}{\hbar^2\kappa^2}\sum\frac{1}{\lambda_n}\chi_n(x)\bar{\chi}_n(-x) \quad (56)$$

$$\lambda_n = \sqrt{1 - \frac{2mE_n}{\hbar^2/\kappa^2}}. \quad (57)$$

Consider, now, the much simpler case where $\hbar^2k^2 \gg E_{n'}$. In that case, $\lambda_n \to 1$, and thus $Z \to 0$ and $R \to 1 + (\alpha\epsilon m/\hbar k)^2$, finally, the transmission coefficients can be written as:

$$s_1 = \tau\ \chi_{n'}(x) \quad (58)$$
$$s_2 = \tau\ \chi_{n'}(-x) \quad (59)$$

where:

$$\tau = \frac{\alpha\epsilon m}{i\hbar k}\frac{\epsilon^{1/2}}{1 + \left(\frac{\alpha\epsilon m}{\hbar k}\right)^2}. \quad (60)$$

B.2 Interference fringes

For an incident state in the waveguide of the form $|\chi_n\rangle \otimes |k\rangle$, where $|\chi_n\rangle$ is an eigenstate of $\hat{H}_{\text{transv.}}$ and $|k\rangle$ a plane wave in the z-direction, one associates the transmitted states in the lead, written:

$$\begin{pmatrix} s_{1n}|k_1\rangle \\ s_{2n}|k_2\rangle \end{pmatrix} \tag{61}$$

where $|k_1\rangle$, $|k_2\rangle$ are plane waves and $(s_{1n}, s_{2n}) \in \mathbf{C}^2$ are transmission coefficients which may depend on n and k. Thus, by the separation principle, if the particle in the guide is described by a density operator:

$$\hat{\sigma} \otimes |k\rangle\langle k| = \left(\sum_{n,n'} \sigma_{nn'} |\chi_n\rangle\langle\chi_{n'}|\right) \otimes |k\rangle\langle k|$$

the transmitted state is described by the density operator:

$$\hat{w} = |\tau|^2 \begin{pmatrix} \sigma(x,x)\,|k_1\rangle\langle k_1| & \sigma(x,-x)\,|k_1\rangle\langle k_2| \\ \sigma(-x,x)\,|k_2\rangle\langle k_1| & \sigma(-x,-x)\,|k_2\rangle\langle k_2| \end{pmatrix}.$$

The probability that the particle be found in the cross-point of the two leads, after a measure is:

$$I(\phi) = \langle z_1|\langle z_2|\hat{w}|z_1\rangle|z_2\rangle \tag{62}$$

$$= |\tau|^2 \Big[\sigma(x,x) + \sigma(-x,-x) \tag{63}$$

$$+\sigma(-x,x)e^{i\phi} + \sigma(x,-x)e^{-i\phi}\Big]. \tag{64}$$

By replacing $\sigma(x,x')$ with its value form equation 50 we can explicitly evaluate $I(\phi)$:

$$I(\phi) = 2|\tau|^2\sigma(x,x) \times \left(1 + e^{-x^2/2\xi^2}\cos\phi\right)$$

$$\text{where} \quad \xi^2 = \frac{\langle\hat{q}^2\rangle\hbar^2}{4\langle\hat{p}^2\rangle\langle\hat{q}^2\rangle - \hbar^2}.$$

By using equation 41 and equation 42 we see that ξ is related to the effective temperature and to the effective mass in the following way:

$$\xi^2 = \frac{\hbar}{2\tilde{m}\Omega} \times \frac{1}{1 - \text{th}\,\frac{\hbar\Omega}{2k\tilde{T}}}. \tag{65}$$

Since, in appendix A, $\tilde{T}$ and $\tilde{m}$ were written as functions of the internal parameters of the environment, we can do either with ξ.

References

1. D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, H.D. Zeh, *Decoherence and the appearance of a classical world in quantum theory* (Springer, 1996)
2. P. Milonni, *The quantum vacuum: an introduction to quantum electrodynamics* (Academic press, 1994)
3. Y. Imry, *Introduction to mesoscopic physics* (Oxford University Press, 2002)
4. P. Mohanty, E.M.Q. Jariwala, R.A. Webb, Phys. Rev. Lett. **78**, 3366 (1997)
5. B.L. Altshuler, A.G. Aronov, D.E. Khmelnitsky, J. Phys. C: Solid State Phys. **15**, 7367 (1982)
6. D.S. Golubev, A.D. Zaikin, Phys. Rev. Lett. **81**, 1074 (1998)
7. L. Aleiner, B.L. Altshuler, M.E. Gershenson, Phys. Rev. Lett. **82**, 3190 (1999)
8. J. von Delft, J. Phys. Soc. Jpn. Suppl. A **72**, 24 (2003); *Influence functional for decoherence of interacting electrons in disordered conductors*, (unpublished)
9. F.W.J. Hekking, L.I. Glazman, Phys. Rev. B **55** 10 (1997)
10. A.O. Caldeira, A.J. Leggett, Phys. Rev. A **31**, 1059 (1985)
11. K. Nagaev, M. Büttiker, Europhys. Lett. **58**, 475 (2002)
12. A. Jordan, M. Büttiker, Phys. Rev. Lett. **92**, 247901 (2004)
13. H. Grabert, U. Weiss, P. Tanker, Z. Phys. B **55**, 87 (1984)
14. H. Grabert, P. Schramm, G.-L. Ingold, Phys. Rep. **168**, 115 (1988)
15. L.I. Glazman, G.B. Lesovik, D.E. Khmel'nitskii, R.I. Shekhter, JETP Lett. **48**, 238 (1988); Pis'ma Zh. Eksp. Teor. Fiz. **48**, No. 4, 218–220 (1988)
16. P. Cedraschi, V.V. Ponomarenko, M. Büttiker, Phys. Rev. Lett. **84**, 346 (2000)
17. A.O. Caldeira, A.J. Leggett, An. Phys. **149**, 374 (1983)
18. G.W. Ford, J.T. Lewis, R.F. O'Connell, *Quantum Langevin equation*, Phys. Rev. A **37**, 4419 (1988)
19. U. Weiss, *Quantum dissipative systems* (World scientific 2001)
20. A. Ratchov, *Evolution of a small system coupled to a zero-temperature environment*, in preparation (unpublished)
21. S. Albeverio, F. Haake, P. Kurasov, M. Kuś, P. Šeba J. Math. Phys. **37**, 4888 (1996)
22. J.J. Sakurai, *Modern quantum mechanics* (Addison–Wesley 1994)
23. P. Exner, *Open quantum systems and path integrals* (D. Reidel Publishers, 2002)
24. U. Fano, Phys. Rev. **124**, 1866 (1961)
25. C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Atom–photon interaction: basic process and applications* (Wiley-Interscience 1998)
26. F. Marquardt, D.S. Golubev, Phys. Rev. Lett. **93**, 130404 (2004)
27. C. Schroll, W. Belzig, C. Bruder, Phys. Rev. A **68**, 043618 (2003)
28. S. Datta, *Electronic transport in mesoscopic systems* (Cambridge University Press, 2004)
29. P. Ullersma, Physica **32**, 27 (1966)
30. E.B. Davies, Comm. Math. Phys. **33**, 171 (1973)